

Subject : Discrete mathematical Structure

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Branch - CS and IT

Semester - 2<sup>nd</sup>

Max. Marks - 10

Do any four questions : Each question carry equal Marks.

Q-1 Show that the  $(P \wedge Q) \rightarrow (P \vee Q)$  is a tautology.

Q-2 Make a truth table for each of the following

(i)  $(P \vee Q) \wedge r$

(ii)  $(\sim P \vee Q) \vee \sim r$

Q-3 Write contrapositive, converse and inverse of the statement

"The home team wins whenever it is raining."

Q-4 Show that the following argument is valid.

"If I try hard and have talent then I will become a musician. If I become a musician then I will be happy. Therefore I will not be happy then either I do not try hard or have no talent."

Q-5 Verify the distributive laws by use of truth table

~~$P \vee (Q \wedge R) \equiv (P \vee Q) \wedge (P \vee R)$~~   
 $P \vee (Q \wedge R) \equiv (P \vee Q) \wedge (P \vee R)$

Answers to Key

Ans  $\rightarrow 1$   $(P \wedge q) \rightarrow (P \vee q) \equiv \sim(P \wedge q) \vee (P \vee q) \quad [\because P \rightarrow q \equiv \sim P \vee q]$

$$\equiv (\sim P \vee \sim q) \vee (P \vee q) \quad [\text{Demorgan law}]$$

$$\equiv [(\sim P \vee \sim q) \vee P] \vee q \quad [\text{by associative law}]$$

$$\equiv [\sim P \vee (\sim q \vee P)] \vee q \quad [\text{by associative law}]$$

$$\equiv [\sim P \vee (P \vee \sim q)] \vee q \quad [\text{by commutative law}]$$

$$\equiv [(\sim P \vee P) \vee (\sim q \vee q)]$$

$$\equiv T \vee T$$

$$\equiv T$$

Ans  $\rightarrow 2$  Truth table of  $(P \vee q) \wedge r$

P	q	r	$P \vee q$	$(P \vee q) \wedge r$
T	T	T	T	T
T	T	F	T	F
T	F	T	T	T
F	T	T	T	T
T	F	F	T	F
F	T	F	T	F
F	F	T	F	F
F	F	F	F	F

(ii) Truth table of  $(\sim p \vee q) \wedge \sim r$

$p$	$q$	$r$	$\sim p$	$\sim p \vee q$	$\sim r$	$(\sim p \vee q) \wedge \sim r$
T	T	T	F	T	F	F
T	T	F	F	T	T	T
T	F	T	F	F	F	F
F	T	T	T	T	F	F
T	F	F	F	F	T	F
F	T	F	T	T	T	T
F	F	T	T	T	F	F
F	F	F	T	T	T	T

Ans-3: The given statement can be rewritten as

"If it is raining then the home team wins."

Let  $p$ : It is raining

$q$ : The home team wins

Then the statement is of the form  $p \rightarrow q$

(a) Its contrapositive is  $\sim q \rightarrow \sim p$  i.e

"If the home team does not win then it is not raining."

(b) Its converse is  $q \rightarrow p$  i.e

"If the home team wins then it is raining"

(c) Its inverse is  $\sim p \rightarrow \sim q$  i.e

"If it is not raining then the home team does not win."

Ans 4: Let  $p$ : I try hard

$q$ : I have talent

$r$ : I become a musician

$s$ : I will be happy

Premises  $\rightarrow p \wedge q \rightarrow r$

$r \rightarrow s$

conclusion  $\rightarrow \therefore \sim s \rightarrow \sim p \vee \sim q$

S.No.	Step	Reason
1	$p \wedge q \rightarrow r$	hypothesis
2	$r \rightarrow s$	hypothesis
3.	$p \wedge q \rightarrow s$	by hypothetical Syllogisms
4.	$\sim s \rightarrow \sim(p \wedge q)$	by contrapositive
5.	$\sim s \rightarrow \sim p \vee \sim q$	by demorgan law

Hence the argument is valid.

Ans - 5

$p$	$q$	$r$	$q \wedge r$	$p \vee (q \wedge r)$	$p \vee q$	$p \vee r$	$(p \vee q) \wedge (p \vee r)$
T	T	T	T	T	T	T	T
T	T	F	F	T	T	T	T
T	F	T	F	T	T	T	T
F	T	T	T	T	T	T	T
T	F	F	F	T	T	F	F
F	T	F	F	F	T	F	F
F	F	T	F	F	F	T	F
F	F	F	F	F	F	F	F